Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

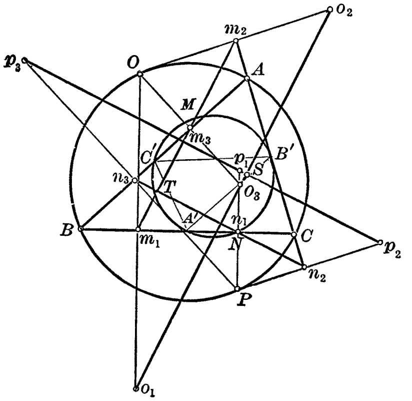
JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

2776 [1919, 267]. Proposed by C. P. SOUSLEY, State College, Penn.

Prove by elementary geometry that the Wallace lines of the extremities of any diameter of the circumscribed circle of a triangle intersect at right angles on the nine-point circle of the triangle.

I. Solution by F. V. Morley, Johns Hopkins University.

Let ABC be the given triangle, O and P the ends of any diameter of the circumcircle. Let p_1, p_2, p_3, be the images of P in the sides of the triangle; then $p_1p_2p_3$ is the *line of images* of P, and $n_1n_2n_3$ the corresponding Wallace line. And similarly for O.



Any line of images passes through S, the orthocenter. For PS is known to be bisected by the Wallace line $n_2n_1n_3$[1] (Casey, *Sequel to Euclid*, second edition, 1882, p. 36). Hence, S is on $p_2p_1p_3$. Similarly, S is on any line of images (l) of a point (p) on the circumcircle.

Consider p as moving round the circle. Then l pivots on S. The angle BSC is the supplement of the angle A, and therefore the image of the circumcircle in BC will pass through S. As p moves through an arc $2x$ along the circumcircle, its image p_1 will move through an equal arc along the circle BSC. The line p_1S, namely l, will rotate about S through the angle x. The rotation of l is in the contrary sense to the motion of p, and of half the measure. Hence, when p moves through an angle π, as from P to O, l moves through the angle $\pi/2$. Therefore $p_1p_2p_3 \perp o_1o_2o_3$.

It follows that the Wallace lines for P and O are perpendicular, for they are parallel to the lines of images.

Now S is a center of similitude of the circumcircle and the nine-point circle (Casey, *l.c.*, p. 105). If M and N are the points in which OS and PS cut the nine-point circle, it follows that MN is parallel to OP, of half its length, and hence a diameter of the nine-point circle. The perpendicular lines $m_1m_3m_2$ and $n_2n_1n_3$, passing through M and N, will therefore meet in T on the nine-point circle.

II. Tucker's Solution, Remarks, and Historical Notes. By R. C. Archibald, Brown University.

The result of this problem was announced, without proof, by Steiner in a memoir read before the Berlin Academy in 1856 and published the following year in *Journal für die reine und angewandte Mathematik* (Crelle), vol. 53, pages 231–237. It was here also that Steiner stated that the envelope of the Wallace lines of a triangle was a three cusped hypocycloid tangent to its sides, and with center at the center of the nine point circle. Hence, from the result of our problem under discussion, it was remarked by Steiner that the orthoptic locus of this curve of the fourth degree and third class, as in the case of a conic section, is a circle (in part, at least). This memoir of Steiner was abridged, translated into English, and published in *Mathematical Questions with their Solutions from the Educational Times*, vol. 3, 1865, pp. 97–100.

Many solutions of the problem proposed by Dr. Sousley have been published. Three, including a brief one by R. Tucker, were published in the volume of *Mathematical Questions* mentioned above, pages 58–59. Others are: by Le Bel and Talayrach, *Nouvelles annales de mathématique*, April, 1865, pp. 177–178; by E. Lemoine in *Journal de mathématiques élémentaires* (Bourget), 1883, pp. 246–247; by Goffart in *Nouvelles annales de mathématique*, August, 1884, pp. 397–399; and by Weill in *Journal de mathématiques spéciales*, 1884, pp. 15–16.

Tucker's solutions is as follows:

"Since Om_1B, Om_3B, Pn_3B, Pn_1B are all right angles Om_1m_3, Pn_3n_1 are equal respectively to OBm_3, PBn_1; and OBP is a right angle; therefore m_3Tn_1 is a right angle.

Again let A', C' be the middle points of BC, BA and therefore also of m_1n_1, n_3m_3; then the angles $A'Tn_1$, $C'Tn_3$ are equal respectively to $A'n_1T$, $C'n_3T$, consequently the angle $A'TC'$ is supplemental to the angle ABC or $C'B'A'$, B' being the middle point of AC; hence T lies on the circle through A', B', C', that is to say, on the nine-point circle of the triangle ABC."

---

[1] This result seems to have been first formulated by Retsin, *Nouvelles annales de mathématiques*, vol. 28, 1869, p. 530—Editor.

In *The Mathematician*, vol. 2, pp. 289–290, July 1847, Fenwick discussed the nine-point conic[1] which may be defined as follows: Given a triangle ABC and a point P in its plane, a conic can be drawn through the following nine points: (1) The middle points of the sides of the triangle; (2) the middle points of the lines joining P to the vertices of the triangle; (3) the points where these last named lines cut the sides of the triangle. Fenwick proved that this conic is the locus of the center of the conics passing through the four points A, B, C, and P; when P is the orthocenter of the triangle the conic is a rectangular hyperbola. Brianchon and Poncelet proved (*Annales de mathématiques pures et appliquées* (Gergonne), tome 11, pp. 205, 213, January, 1821) that every rectangular hyperbola through A, B, and C must also pass through the orthocenter and that the locus of the centers of such hyperbolas is the nine-point circle of the triangle. The asymptotes for any one of the hyperbolas are the pair of Wallace lines corresponding to the ends of a certain diameter of the circumscribed circle. (Steiner, *l.c.*, cf. also Weill, *Journal de mathématiques spéciales*, 1884, p. 16.) When this diameter passes through the center of the inscribed circle we have that the corresponding Wallace lines intersect at the point of contact of the nine-point and inscribed circles (Neuberg, *Mathesis*, 1893, p. 86). The equilateral hyperbola with this point as center has been called the *hyperbola of Feuerbach* (Neuberg, *l.c.*) since the theorem that the nine-point circle of a triangle is tangent to the inscribed and the three escribed circle was enunciated and proved by Feuerbach in a publication of 1822. I have not seen a reference to the three similar hyperbolas corresponding to diameters through the centers of the escribed circles.

Corresponding to the diameter through the orthocenter (that is, Euler's line) we have the *hyperbola of Jeřábek* (Jeřábek, *Mathesis*, 1888, pp. 81–84), and to the diameter through the Lemoine point,[2] we have the hyperbola called by Neuberg (*Journal de mathématiques spéciales*, 1886, p. 73) *hyperbola of Kiepert* (*Nouvelles annales de mathématique*, 1869, pp. 40–42.).

A more general form of Dr. Sousley's problem was given by Weill (*Journal de mathématiques spéciales*, 1884, p. 13): If m and m' are the Wallace lines of any two points M and M' on the circumcircle, then the angles between m and m' are equal to one or other of the angles under which the line segment MM' is seen from a point of the circumcircle.

Also solved by P. J. DA CUNHA, A. PELLETIER, and the Proposer.

NOTES AND NEWS.

EDITED BY E. J. MOULTON, Northwestern University, Evanston, Ill.

Dr. W. H. WILSON, of the Massachusetts Institute of Technology, has been instructor in mathematics at the University of Iowa since last September.

At the University of Oklahoma Associate Professor E. P. R. DUVAL resigned, his resignation to become effective at the end of the first semester, and is engaged in fruit growing at Springdale, Arkansas. Mr. E. E. COWAN has been appointed instructor in mathematics.

At Yale University Assistant Professor W. R. LONGLEY has been promoted to a full professorship, and Instructor J. K. WHITTEMORE to an assistant professorship of mathematics.

---

[1] This name seems to be due to Beltrami, 1863, "Intorno alle coniche dei nove punti e ad alcune questioni che ne dipendone," *Memorie dell'accademia delle scienze dell' Istituto di Bologna*, serie 2, vol. 2 (1862), pp. 361–395; also *Opere matematiche di Eugenio Beltrami*, vol. 1, 1902, pp. 45–72.

[2] This is the point of intersection of the symmedians of a triangle, that is, the lines through the vertices of the triangle symmetric to the medians through those vertices, with respect to the corresponding angle bisectors.